GCSE Maths – Ratio, Proportion and Rates of Change

Compound Growth and Decay

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of compound growth and decay questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

The population of 250 rabbits in a field increases by 3% each year. How many rabbits will there be after 4 years?

Step 1: Find values for N_0 and t for use in the formula $N = N_0 \times \left(1 + \frac{percentage}{100}\right)^n$.

$$N_0 = 250$$
$$t = 4$$

Step 2: Substitute into the formula to calculate the value of N.

$$N = N_0 \times \left(1 + \frac{percentage}{100}\right)^n$$

$$N = 250 \times \left(1 + \frac{3}{100}\right)^4$$

$$N = 250 \times 1.03^4 = 281.377 \ldots$$

Step 3: Form a conclusion.

To the nearest whole number there will be 281 rabbits in the field after 4 years.

Guided Example

The population of a beehive is currently 2000, however due to some circumstances the population is increasing by 7% a year. What is the population of the beehive after 10 years to 3 significant figures?

Step 1: Find values for N_0 and t for use in the formula $N = N_0 \times \left(1 + \frac{percentage}{100}\right)^n$.

$N_0 = 2000$
$t = 10$

Step 2: Substitute into the formula to calculate the value of N.

$$N = 2000 \times \left(1 + \frac{7}{100}\right)^{10}$$
$$= 2000 \times 1.07^{10}$$
$$= 3934.3\ldots$$

Step 3: Form a conclusion.

To 3sf there would be 3930 bees in 10years.

3930

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Red Squirrels are entering the UK at a rate of 5.2% a year. Currently there are 590 red squirrels in the UK. What is the expected number red squirrels after 6 years?

Formula : $N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$

$N_0 = 590$
$n = 6$
$\% = 5.2$

$N = 590 \times \left(1 + \frac{5.2}{100}\right)^6$
$= 590 \times 1.052^6$
$= 799.735\ldots$
(nearest whole number)

After 6 years, the expected number of red squirrels is 800.

2. UK retirees are migrating to holiday homes in Spain. Every year, 2.3% of UK residents move to Spain. In 2021, there are 2700 UK retirees. How many retirees will there be in 2027?

Formula : $N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$

$N_0 = 2700$
$n =$ from 2021 to 2027
$= 2027 - 2021 = 6$
$\% = 2.3$

$N = 2700 \times \left(1 + \frac{2.3}{100}\right)^6$
$= 2700 \times 1.023^6$
$= 3094.69\ldots$
(nearest whole number)

In 2027, the expected number of UK retirees in Spain is 3095.

3. On Tuesday 30,000 people tested positive for Covid-19. The health secretary estimates the cases increases 4.7% a day. How many more people have test positive on Sunday than Tuesday?

Calculate positive cases reported on Tuesday.

Formula : $N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$

$N_0 = 30{,}000$

$n =$ Tuesday to Sunday

$= 5$ days

$\% = 4.7$

$N = 30000 \times \left(1 + \frac{4.7}{100}\right)$

$= 30{,}000 \times 1.047^5$

$= 37744.58\ldots$

Calculate the difference

$37745 - 30000 = 7745$ (nearest whole number)

There were 7745 more cases.

4. In 2010, the population of trout in a fishery is 4000. In 2016, the new population is 5642. What is the population growth rate?

Use the formula to rearrange to find the growth rate.

Formula : $N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$

$N_0 = 4000$

$n =$ from 2010 to 2016

$= 2016 - 2010 = 6$

$N = 5642$

$\% =$ unknown.

$5642 = 4000 \times \left(1 + \frac{\%}{100}\right)^6$

$\div 4000$

$1.4105 = \left(1 + \frac{\%}{100}\right)^6$

$\sqrt[6]{\ }$

$1.058999 = 1 + \frac{\%}{100}$

-1

$0.058999 = \frac{\%}{100}$

$\times 100$

$5.8999 = \%$

To 3 significant figures, the growth rate is 5.90%.

Section B

Worked Example

The population of 10,000 rabbits in a field decreases by 10% each year due to food shortages. How many rabbits will there be after 4 years?

Step 1: Find values for N_0 and t for use in the formula $N = N_0 \times \left(1 - \frac{percentage}{100}\right)^n$.

$$N_0 = 10{,}000$$
$$t = 4$$

Step 2: Substitute into the formula to calculate the value of N

$$N = N_0 \times \left(1 - \frac{percentage}{100}\right)^n$$

$$N = 10{,}000 \times \left(1 - \frac{10}{100}\right)^4$$

$$N = 10{,}000 \times 0.9^4 = 6561$$

Step 3: Form a conclusion.

There will be 6561 rabbits in the field after 4 years.

Guided Example

The value of a gold necklace is depreciating at a rate of 0.04% a year. Currently it is worth £13,000. What will the value be after 7 years?

Step 1: Find values for N_0 and t for use in the formula $N = N_0 \times \left(1 - \frac{percentage}{100}\right)^n$.

$N_0 = 13{,}000$
$n = 7$

Step 2: Substitute into the equation to calculate the value of N.

$$N = 13000 \times \left(1 - \frac{0.04}{100}\right)^7$$
$$= 13000 \times 0.9996^7$$
$$= 12{,}963.64$$

(nearest pence)

Step 3: Form a conclusion.

After 7 years, the value of the Gold Necklace will be worth £12,963.64

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Water in a tank is leaking at a rate of 5.5% a second. The tank is filled up with 6 l of water. How much water is left after 8 seconds? Give your answer in millilitres.

Formula : $N = N_0 \times \left(1 - \frac{\text{percentage}}{100}\right)^n$

$N_0 = 6$
$n = 8$
$\% = 5.5$

$N = 6 \times \left(1 - \frac{5.5}{100}\right)^8$
$= 6 \times 0.945^8$
$= 3.815977\ l$

in mL $\xrightarrow{\times 1000}$ 3815.977 mL

To the nearest millilitre there will be 3816mL left.

6. A new car is bought for £15,000. It depreciates by 33% each year. Tim sells his car for the value after 3 years. How much did Tim lose?

Formula : $N = N_0 \times \left(1 - \frac{\text{percentage}}{100}\right)^n$

Calculate how much Tim sells :

$N_0 = 15000$
$n = 3$
$\% = 33$

$N = 15000 \times \left(1 - \frac{33}{100}\right)^3$
$= 15000 \times 0.67^3 = 4511.445$

Calculate loss.

$15,000 - 4511.445 = 10,488.56$ (nearest pound)

Tim loses £10, 488.56

7. A bouncy ball is thrown from a height of 5 m. It bounces at a height 4.5% less than the height before. How many bounces does it take for the ball to be under 1 m of height?

Formula : $N = N_0 \times \left(1 - \frac{\text{percentage}}{100}\right)^n$

Use trial and error to find the number bounces.

$N_0 = 5$

$\% = 4.5$

$n = n$

After 5 bounces:

$N = 5 \times \left(1 - \frac{4.5}{100}\right)^5$

$= 5 \times 0.955^5 = 3.97\ldots$ (above 1m)

After 35 bounces:

$= 5 \times 0.955^{35} = 0.997\ldots$ (below 1m)

34 bounces:

$= 5 \times 0.955^{34} = 1.0449$ (above 1m)

check to see if 34 bounces is above 1m

After 35 bounces, the ball will be under 1m.

8. The value of a car depreciates at the rate x %. In 2020, the value is £21,000. In 2028, the value of the car is approximately £11,255. Find the value of x.

Formula : $N = N_0 \times \left(1 - \frac{\text{percentage}}{100}\right)^n$

Using the formula, rearrange to find x.

$N_0 = £21{,}000$

$N = £11{,}255$

$\% = x$

$n =$ from 2020 to 2028

$= 2028 - 2020 = 8$

$11{,}255 = 21{,}000 \times \left(1 - \frac{x}{100}\right)^8$

$\div 21000$

$0.53595\ldots = \left(1 - \frac{x}{100}\right)^8$

$\sqrt[8]{\ }$

$0.9250\ldots = \left(1 - \frac{x}{100}\right)$

-1

$-0.0750\ldots = \frac{-x}{100}$

$\times -100$

$7.500\ldots = x$

$x = 7.50$ (3 s.f.)

Section C

Worked Example

Hana deposits £800 in a bank that pays 4.5% compound interest a year. Work out the interest paid by the bank in 3 years.

Step 1: Find values for N_0 and t for use in the formula $N = N_0 \times \left(1 + \frac{percentage}{100}\right)^n$.

$$N_0 = 800$$
$$t = 3$$

Step 2: Substitute into the formula to calculate the value of N.

$$N = N_0 \times \left(1 - \frac{percentage}{100}\right)^n$$

$$N = 800 \times \left(1 + \frac{4.5}{100}\right)^3$$

$$N = 800 \times 1.045^3 = 912.9329$$

Step 3: Form a conclusion.

There will be £912.93 in Hana's bank account after 3 years.

Guided Example

Chloe loans £5500 from a bank where the cost of borrowing is 3% per year. Calculate the extra amount of compound interest Chloe pays in 6 years.

Step 1: Find values for N_0 and t for use in the formula $N = N_0 \times \left(1 + \frac{percentage}{100}\right)^n$.

$N = 5500$
$n = 6$

Step 2: Substitute into the formula to calculate the value of N.

$N = 5500 \times \left(1 + \frac{3}{100}\right)^6$
$= 5500 \times 1.03^6 = 6567.2876...$

Step 3: Calculate how much interest this is.

$$Interest\ Paid = New - Original$$
$6567.28... - 5500 = 1067.287...$

Step 4: Form a conclusion.

To the nearest pence, Chloe earns £1067.29 extra interest

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

9. Ethan loans £700 from a bank where the cost of borrowing is 5% per year. Calculate the extra amount of compound interest Ethan pays in 2 years.

Formula : $N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$

$N_0 = 700$, $n = 2$, $\% = 5$

$N = 700 \times \left(1 + \frac{5}{100}\right)^2$

$N = 700 \times 1.05^2$

$= 771.75$

$771.75 - 700$

$= 71.75$

Ethan pays £71.75 extra interest

10. Rhea deposits £1150 in a bank that pays 4% compound interest a year. Work out the interest paid by the bank in 3 years.

Formula : $N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$

$N_0 = 1150$, $n = 3$, $\% = 4$

$N = 1150 \times \left(1 + \frac{4}{100}\right)^3$

$= 1150 \times 1.04^3$

$= 1293.5936$

$1293.5936 - 1150 = 143.5936$

To the nearest pence, Rhea gets £143.59 from the bank

11. Delaney loans £5800 from a bank where the cost of borrowing is 6.7% per year. Calculate the amount of compound interest Delaney pays in 10 years.

Formula : $N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$

$N_0 = 5800$, $n = 10$, $\% = 6.7$

$N = 5800 \times \left(1 + \frac{6.7}{100}\right)^{10}$

$= 5800 \times 1.067^{10}$

$= 11093.59...$

$11093.591 - 5800 = 5293.59196$

To the nearest pence, Delaney pays an extra £5293.59

12. Maya deposits £756 in a bank that pays 2.3% compound interest a year. Work out the interest paid by the bank in 6 years.

Formula : $N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$

$N_0 = 756$, $\% = 2.3$, $n = 6$

$N = 756 \times \left(1 + \frac{2.3}{100}\right)^6$

$= 756 \times 1.023^6$

$= 866.514...$

$866.514.. - 756$

$= 110.514...$

To the nearest pence, Maya earns £110.51